2023-2024 学年高等代数与解析几何 2-1 第二次月考

回忆:zwj

一.解线性方程组$\begin{cases} 2x_1+\lambda x_2-x_3=1 \\ \lambda x_1-x_2+x_3=2 \\ x_1+x_2-x_3=-1 \end{cases}$.

二.设向量组$\alpha_1,\alpha_2,\cdots,\alpha_s$线性无关,且任意$\alpha_i$都可由量组$\beta_1,\beta_2,\cdots,\beta_t$线性表出.证明:存在$\beta_{j_1}$, $\beta_{j_2},\cdots,\beta_{j_{t-s}}$,使得向量组$\alpha_1,\alpha_2,\cdots,\alpha_s,\beta_{j_1},\beta_{j_2},\cdots,\beta_{j_{t-s}}$与向量组$\beta_1,\beta_2,\cdots,\beta_t$等价.

三.已知线性方程组$\begin{cases} a_{11}x_1+a_{12}x_2+\ldots+a_{1n}x_n=0 \\ a_{21}x_1+a_{22}x_2+\ldots+a_{2n}x_n=0 \\ \ldots \\ a_{n1}x_1+a_{n2}x_2+\ldots+a_{nn}x_n=0 \end{cases}$的系数矩阵$A=\left(a_{ij}\right)$满足:(i)$|A|=0$.(ii)某个元素$a_{ij}$的代数余子式$A_{ij}\neq 0$.证明:上述线性方程组的解都能写成如下形式:

$$\lambda\begin{bmatrix} A_{i1} \\ A_{i2} \\ \cdots \\ A_{in} \end{bmatrix}.$$

四.设向量α_1, α_2, $\cdots$, α_n线性无关, $\beta=\sum_{i=1}^{n}\alpha_i$, 且$a_i>0$, $\forall 1\leq i\leq n$.证明: $\beta+a_1\alpha_1$, $\beta+a_2\alpha_2,\cdots,\beta+a_n\alpha_n$也线性无关.

五.已知已下非齐次方程组有非零解:

$$\begin{cases} a_{11}x_1+a_{12}x_2+\ldots+a_{1n}x_n=b_1 \\ a_{21}x_1+a_{22}x_2+\ldots+a_{2n}x_n=b_2 \\ \ldots \\ a_{m1}x_1+a_{m2}x_2+\ldots+a_{mn}x_n=b_m \end{cases}.$$

证明:每个解向量中第k个分量都等于0的充分必要条件是将增广矩阵$\overline{A}$去掉第k列后得到的矩阵秩比$\overline{A}$小.

六.已知向量组

$$\alpha_1=(6,4,1,-1,2),\alpha_2=(1,0,2,3,-4),\alpha_3=(1,4,-9,-16,22),\alpha_3=(7,1,0,-1,3).$$

判断是否存在$\{1,2,3,4\}$的一个排列i_1,i_2,i_3,i_4使得方程

$$x_1\cdot\alpha_{i_1}+x_2\cdot\alpha_{i_2}+x_3\cdot\alpha_{i_3}=\alpha_{i_4}$$

有解,并说明理由.若存在,请找出一个这样的排列.

七.在实数域中解线性方程组

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = 0 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = 0 \\ \cdots \\ a_{s1}x_1 + a_{s2}x_2 + \ldots + a_{sn}x_n = 0 \end{cases}$$

求得一个基础解系为$\eta_1 = \left(b_{11}, b_{12}, \ldots, b_{1n}\right)$,$\eta_2 = \left(b_{21}, b_{22}, \ldots, b_{2n}\right)$,$\cdots$,$\eta_t = \left(b_{t1}, b_{t2}, \ldots, b_{tn}\right)$.证明:方程组

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = 0 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = 0 \\ \cdots \\ a_{s1}x_1 + a_{s2}x_2 + \ldots + a_{sn}x_n = 0 \\ b_{11}x_1 + b_{12}x_2 + \ldots + b_{1n}x_n = 0 \\ \cdots \\ b_{t1}x_1 + b_{t2}x_2 + \ldots + b_{tn}x_n = 0 \end{cases}$$

只有零解.