统计与数据科学学院本科生 2022——2023 学年第 一 学期《时间序列分析》课程期末考试试卷（A 卷）

专业：　　　　年级：　　　　学号：　　　　姓名：　　　　成绩:

草 稿 区

得 分

、一 、（本题共 10 分，每小题 5 分）

a) Give the definitions of strictly stationary（强平稳）time series and weakly stationary（弱平稳）time series. Why the stationary condition is important?

b) Give the definition of an integrated autoregressive moving average model (ARIMA). What are the sufficient and necessary conditions for an ARIMA model to be causal and invertible?

草 稿 区

得 分

、 二、 （本题共 20 分，每小题 5 分）

Consider the random walk with drift model, $x_t = \delta + x_{t-1} + w_t, t = 1,2,\ldots$, with $x_0 = 0$, where w_t is white noise with mean zero and constant variance σ_w^2.

a) Show that the model can be written as $x_t = t\delta + \sum_{i=1}^{t} w_i$;

b) What are the mean function and auto-covariance function of x_t? Is x_t stationary?;

c) Show the auto-correlation function $\rho_x(t-1, t) = \sqrt{\frac{t-1}{t}} \to 1, as\ t \to \infty$.

d) Suggest a transform to make x_t stationary.

草 稿 区

得 分

、三 、（本题共 20 分，每小题 10 分）

Suppose x_t follows an ARMA(1,1) model with zero mean, $x_t = \phi x_{t-1} + \theta w_{t-1} + w_t$.

a) Obtain the auto-covariance function and auto-correlation function of x_t ;

b) What is the stationary solution of the (causal) ARMA(1,1) model? Obtain the coefficients for w_{t-k}, $k = 0, 1, 2, ...$ in the stationary solution.

草 稿 区

得 分

、四 、（本题共 20 分，每小题 10 分）

a) Describe the steps for fitting an appropriate ARIMA model to a time series;

b) For a general stationary time series x_t with mean zero, derive the prediction equations for obtaining the best (minimum-mean-squared-error) linear predictor of x_{n+m} based on n observations, $\{x_1, x_2, \ldots, x_n\}$.

草 稿 区

得 分

、五、 （本题共 20 分，每小题 10 分）

Consider an AR(1) model, $x_t - \mu = \phi(x_{t-1} - \mu) + w_t$, where μ is a constant mean and w_t is Gaussian white noise with mean zero and variance σ_w^2. Suppose we observe $x_1, x_2, \ldots, x_n$ from the x_t series.

a) Show the Yule-Walker equations for estimating ϕ and σ_w^2. Obtain the corresponding estimate （矩估计）for ϕ based on n observations;

b) Show the (Gaussian) conditional log-likelihood function (条件对数似然函数) given x_1 for parameter estimation. Obtain the estimate of ϕ based on the conditional log-likelihood function.

草 稿 区

得 分

、六、 （本题共 10 分，每小题 5 分）

Given data $x_1, x_2, \ldots, x_n$ with a constant mean μ, define the discrete Fourier transform for a given frequency ω_j: $d(\omega_j) = n^{-1/2} \sum_{t=1}^{n} x_t e^{-2\pi i \omega_j t}$, where $i = \sqrt{-1}$, $\omega_j = \frac{j}{n}$, and j=0, 1, 2, ..., n-1.

a) What is the definition of periodogram, $I(\omega_j)$? How to interpret it?

b) Show that for $j \neq 0$,

$$I(\omega_j) = \sum_{h=-(n-1)}^{n-1} \hat{\gamma}(h) \exp(-2\pi i \omega_j h)$$

,

where $\hat{\gamma}(h)$ is the sample auto-covariance function. What is the implication of b) for explaining the periodogram $I(\omega_j)$?