

2020-2021 DIFFERENTIAL MANIFOLD

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1. Write down the definitions of following terms.

Manifolds Embedded Submanifold Riemann Manifold Cross Section Curvature

2. Prove the cotangent bundle T^*M is a smooth manifold with dimension of $2m$.
3. Calculate three Lie brackets.

$$[X, Y] = \left[X^i \frac{\partial}{\partial u^i}, Y^j \frac{\partial}{\partial u^j} \right] = [X(Y^i) - Y(X^i)] \frac{\partial}{\partial u^i}$$

4. Prove $\frac{xdx+ydy}{x^2+y^2}$ is a closed form, and decide whether it is an exact form.
5. Prove the manifold with Riemann metric $G = \frac{dx \otimes dx + dy \otimes dy + dz \otimes dz}{1+K(x^2+y^2+z^2)^2}$ has constant curvature.
6. Prove there exists singular point on the tangent bundle of any even order sphere.

Note: The exam is in Chinese :-)